Enabling the Willing: Consumer Rebates for Durable Goods

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ABSTRACT

In financing the purchase of major durable goods by loans, consumers have to put down a significant down payment. Some consumers who are willing to buy a new car are unable to do so because of their inability to provide a down payment. We develop a model in which a durable goods manufacturer recognizes this problem and gives consumers a cash rebate that consumers can use for the down payment. We derive equilibrium prices, quantities and profits for the manufacturer that sells its product through a retailer. We find among other things, that under certain conditions, the manufacturer with the lower durability product is more likely to give cash rebates. In addition, when such rebates are given, the manufacturer with the lower durability product offer a greater cash rebate. We empirically test some of our equilibrium results using a proprietary data base that includes all the customer cash promotions given by the three U.S. car manufacturers and the four major Japanese manufacturers for the period 1992-1997. We find strong support for our model predictions.
1. INTRODUCTION

Major durable goods such as automobiles, furniture and home entertainment systems are expensive relative to most consumers’ income. Consequently, these purchases are normally financed, requiring the consumer to have enough liquid assets to make a significant down payment (typically in the range of 10-20% of the purchase price). For durables like automobiles where a second hand market exists, the purchasers often want to use the proceeds from their existing (used) car as this liquid asset. This requires the potential customers to have enough equity in the durable to make the down payment. Interestingly, approximately 40% of all car owners are “upside down”, i.e., still owe more money on their car than the current market value of the car (USA Today, 2003). If these consumers want to replace their existing cars with new cars, they not only have to come up with a down payment, but also must pay off the amount they are still upside down. This inability to use the used car for the down payment is often a barrier that stops them from purchasing a new car.

We develop and analyze a model in which a durable goods manufacturer gives cash rebates (often called customer cash by the trade) in order to help consumers solve their liquidity problems, thereby stimulating the new car demand. We chose to study this problem of cash rebates even though numerous prior studies look at retail promotions. We do this for a number of reasons. First, customer rebates in the automobile industry alone represent marketing expenses that exceed $3 billion dollars per year. Second, to the best of our knowledge no one has modeled a situation where a) a manufacturer directly gives a rebate to consumers, b) an independent dealer sets the retail price and c) any increased sales that come from these rebates produces second hand durables that compete with new durable sales. Finally, even though we use the automobile industry as our model for studying this marketing practice, we believe our results can be generalized to other industries that are characterized by similar stylized facts.

Our study is composed of two parts. After briefly reviewing the literature on price promotions, we derive a demand structure for new and used expensive durables distributed through a third party outlet (e.g., a franchised dealer). This demand structure is a function of the

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1 For example, a recent report showed that 91% of all new car buyers planned to finance their next car purchase. (Research Alert, November 1997)
quality of the new and used car, the retail prices of the two competing cars as well as size of the customer rebate. We then use this demand structure to solve for equilibrium prices, quantities and profits using a game theoretic model of an industry composed of one durable manufacturer and one franchised dealer. This allows us to derive a series of implications on how profits, prices, and sales vary with some of the parameters of our model. For example, we show that a manufacturer of a lower quality durable is more likely to use cash rebates and give deeper discounts than a higher quality durable manufacturer. We also show that the effective price to consumers raises when manufacturers give customer cash. In the second part of our study, we empirically test some of our equilibrium results using a proprietary data base that includes all the cash rebates promotions given in the United States by the three U.S. car manufacturers and the four major Japanese manufacturers for the period 1992-1997. We find strong support for our model predictions.

We believe this paper makes a number of substantive and theoretical contributions. First, it offers a new explanation for why a manufacturer might want to give a cash rebate directly to a consumer. Specifically we show this pricing action allows the manufacturer to increase sales by shifting the demand function outward instead of sliding down the demand curve. Second, we explicitly allow for the empirical fact that not everyone who is willing to pay for a durable has the ability to pay for this durable. We do this by augment the often used construct “willingness-to pay” with an additional construct: ability-to-pay. We believe this new approach can be applied to numerous durable goods markets. Third, we provide a series of testable hypotheses concerning the profits and prices associated with cash rebates. For example, we show that the margins decrease for both the manufacturer and retailer although both derive increased profits from the rebate. Fourth, we provide empirical evidence that strongly supports three of our major results. We discuss all these issues in more depth after we derive our results.

We start our analysis by noting that cash rebates are given with the idea of increasing demand. Conceptually, this increased demand can either come from the firm sliding down a fixed demand curve, i.e., any increase in the quantity sold comes from a lower effective price after the rebate, or from the firm shifting the demand curve outwards, i.e., the firm gets more sales for any given effective price. In the former case, new consumers enter the market because

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2 Note that we could have equivalently developed a model where the manufacturer gives lower APR rates, thereby enabling consumers who lack the ability to make the monthly payments at the higher interest rate to now make the
the effective retail price is lower. In the latter case, they enter the market because receiving cash rebates provides them with value, independent of getting a lower effective retail price. Determining which of the two ways cash rebates impact demand the most depends on the industry we study. **In our case we note that** when the auto manufacturer provides customers with a direct cash payment some consumers who otherwise lack the ability to purchase a car now are able to enter the market, thereby shifting the demand curve outward. Specifically, cash rebates allow those customers who have a high valuation for the product but who lack the ability-to-pay because of the stringent down payment terms, now to buy the product even if the effective purchase price remains the same. In addition, unless the retailer determines it is in his best interest to increase the retail price more than the size of the rebate, the effective retail price will decrease, allowing the firm to slide down the demand curve. These stylized facts lead us to postulate a model where consumers are heterogeneous with respect to a) their ability to come up with the necessary down payment and b) their willingness to pay. With this as background, we next review prior work on promotions and then present our model of buyer behavior and derive the appropriate demand functions.

### 2. LITERATURE REVIEW

There is a rich literature on price promotions. Blattberg, et al. (1995) provide a number of empirical generalizations concerning the effects of price promotions. However, almost all of these generalizations come from studies of consumer non-durables. One finding that has been replicated numerous times is that temporary retail price reductions substantially increase the sales of the consumer non-durable brand being promoted. This increase in sales also was observed by Pauwels, et al. (2002) for automobiles who report short term (one week) and long term (one to two months) increases in sales associated with *unexpected* retail price reductions. Interestingly, the above discussed findings are mixed with respect to the degree to which these increased sales come from other brands (i.e., brand switching) or increases due to category expansion.

Given the large impact of price promotions on brand sales, it is not surprising that a number of researchers have offered explanations for this occurrence. A few of these explanations are based on monopoly models and thus do not require brand switching. Blattberg,
et al. (1981) postulate that retailers offer price promotions to reduce their inventory carrying costs by transferring the holding cost to the consumer. Jeuland and Narasimhan (1985) expand on this idea by building a model where consumers with higher holding costs also have higher consumption rates. In this case, promotions are given to these high-cost, high-consumption customers to get them to stockpile the promoted brand thereby increasing consumption. Gerstner and Hess (1995) assume two distinct segments of consumers who vary in terms of willingness to pay. They show that a manufacturer who runs pull promotions targeted at price-conscious consumers can help coordinate channels, i.e., reduced the double marginalization problem, by getting the retailer to also target this price conscious segment and therefore lowering his price.

Other researchers formulate models that rely on competition across brands. In marketing, four of the relevant studies are Rao (1991), Narasimhan (1988), Lal (1990) and Raju et al. (1990). All of these models look at price promotions within a non-durable setting. Moreover, they do not reflect the fact that the manufacturer distributes the product through an independent retailer who has control over the retail price. Lal (1990) argues that national brands alternately promote to limit the sales of a store brand. Using a three-brand model, Rao (1991) shows that only national brands (as opposed to store brands) will promote. Narasimhan (1988) shows that in a two-brand market, the depth and frequency of a price promotion is a function of the size of the switcher pool and the switchers’ pre-price preference for a particular brand. Raju et al. (1990) assume all the sales increases come at the expense of the competing brand and show that the stronger of the two brands (i.e., the brand with the larger loyalty pool) will promote less.

There is also a large literature on durable goods. However, this literature is primarily focused on addressing the time-consistency problem (see for example, Coase 1972, Bulow 1982, Stokey 1981). A few marketing papers in this stream have looked at channel coordination problems (Jeuland and Shugan, 1983; McGuire and Staelin, 1986; Purohit and Staelin 1994, Purohit (1997) and the relative benefits of leasing and selling (Desai and Purohit 1998, 1999). Only one paper (Bruce, Desai and Staelin, 2003) looks at promotions in a durable industry. However, in this latter case, the authors model promotions given to the trade (i.e., retailer) and not to the consumer.

In summary, none of these studies deals with issues of consumers’ cash constraints. In addition, none of these studies explain why a durable goods manufacturer would give a rebate to consumers in situations where the retailer has direct control of the retail price. Also, none of
these studies capture the fact that increased sales have the secondary effect of reducing second-hand durable prices and thus decreasing new durable prices because of the intra-brand competition between the new and used durables. With this in mind, we next develop a model that captures the three key aspects of the situation of interest, i.e., the existence of a completing second hand market, some consumers needing liquid assets to use to make a down payment, and a distribution system that includes an independent retailer who sets the retail price after observing the size of the rebate.

3. MODEL DEVELOPMENT

Our interest is in cash rebates and their use in the automobile industry. This leads us to model both the new and secondhand markets and the interaction of these two markets. We acknowledge that the automobile industry is competitive and there are numerous product offerings. However, given our interest in the type of manufacturer that is most likely to use customer rebates and how these rebates affect this manufacturer’s profits and prices, we limit the competitive environment to the intra-brand competition between a manufacturer’s new and used cars. We discuss this decision later, but the consequence of this assumption is a model of an industry composed of one durable manufacturer and one franchised dealer. The manufacturer chooses the rebate amount (if any) and the wholesale price. The dealer chooses the retail price.

Our model of consumer behavior needs to reflect key aspects of consumer characteristics. First, we need to recognize that not all consumers have the ability to buy the durable. We do this by assuming consumers are heterogeneous with respect to the amount of liquid assets they have available to make a down payment. Second, we need to allow for consumer heterogeneity with respect to their valuation for the durable, which in turn affects their desire to own the durable. This heterogeneity could reflect the appropriateness of the durable in terms of individual tastes (e.g., color, size, etc.) and/or the consumers’ basic needs (e.g., some consumers must use a car to get to work, others have viable options such as walking, public transportation, etc.). Finally, we must recognize that some consumers have an existing durable that can still meet their needs at least within the time period being analyzed while others do not have a useable durable.

We model these aspects as follows. We assume that $\alpha$ fraction of consumers currently owns a used car and $1 - \alpha$ fraction of consumers does not own any used car. We further assume
that $c_\theta \ (0 < c_\theta < 1)$ fraction of the consumers who owns a used car does not have any liquidity problems and these consumers can buy a new car if they prefer to do so. The remaining $1 - c_\theta$ fraction of used car owners has liquidity problems. Consequently these consumers do not have the ability to buy a new car unless they get access to some liquid assets.

We allow for variation in the consumers’ willingness-to-pay for new and used cars. We do this by assuming consumer $i$’s willingness-to-pay for a car is $\theta_i \phi(\gamma)$ where $\theta_i$ is a consumer-specific parameter representing the consumer’s valuation for the services provided by the car and $\phi(\gamma)$ is a car-specific valuation parameter that depends on the durability of the car ($\gamma$), and on the newness of the car: $\phi(\gamma) = n(\gamma)$ for new cars and $\phi(\gamma) = u(\gamma)$ for used cars. The parameter $\gamma \ (0 \leq \gamma \leq 1)$ represents how well a car holds up with usage and measures the lack of physical deterioration of the car. Since the durability directly affects the services that a used car provides we assume consumers prefer more durability to less when valuating a used car. Thus $u'(\gamma) > 0$. Likewise, since consumers think about the future use of the car when buying a new car, durability also affects consumers’ valuation of a new car, $n(\gamma)$. Again we assume consumers prefer more durability to less. Thus $n'(\gamma) > 0$. Finally, we expect consumers to prefer new cars to old cars except in the limit case when the used car is identical to the new car ($\gamma = 1$), i.e., $n(\gamma) \geq u(\gamma)$, with the equality holding only when $\gamma = 1$.

For the car-owning segment (i.e., the $\alpha$ segment), we make the following assumptions:

1) The consumer-specific valuation parameter $\theta_i$ is uniformly distributed between parameters $\theta_0$ and $\tau$ where $0 < \theta_0 \leq \theta_i \leq \tau$. We assume that $\theta_i$ is large enough to insure that the consumers found it best to buy their last car and $\tau$ represents the upper limit on value.

2) Consumers are distributed uniformly with respect to the amount of liquid assets available for a down payment. Let the degree to which consumers lack the liquid assets needed to make a down payment vary uniformly from zero to $p$, where $p$ represents some upper bound on the need for total liquidity.

3) The consumer’s ability and valuation are independent of each other. Thus, consumers are uniformly distributed over a rectangle where the horizontal axis represents their valuation and the vertical axis represents their ability to pay (See Figure 1).
In order to calculate demand we need to divide these consumers into different segments according to their purchase behavior. Since all these consumers currently own a car, they can either sell their old car and buy a new one, or keep the old car. We determine which action each consumer will take by assuming a consumer will take the action that maximizes the individual’s utility net of price. Let $p_n$ be the retail price of the new car and $R$ be the size of the cash rebate given to buyers of a new car. Finally, let the used car price be $p_u$. Then for consumer $i$ who currently owns a car, the utility associated with the strategy of selling the old car and buying a new one is:

$$u_{ni}^I = n(\gamma)\theta_i - p_n + R + p_u,$$

where the superscript refers to the segment of current car owners. Similarly, the utility for keeping the existing car is

$$u_{ki}^I = u(\gamma)\theta_i.$$

Two aspects of Equations 1 and 2 are of special interest. First the person specific parameter $\theta_i$ impacts the degree to which the individual values the intrinsic value of the durable. Thus individuals with large values of $\theta_i$ derive more utility after purchasing the car for a given price than those with small values of $\theta_i$. Second, we want our model to reflect the two effects of the rebate, $R$. First, it needs to lower the effective price of the new car before trade in, i.e., from $p_n$ to $(p_n - R)$. Second, as $R$ increases, we want more individuals enter the market since they are now able to use the available cash rebate to make the down payment. We capture this second effect by assuming that as $R$ increases so does the proportion of buyers who are able to buy a new car. We operationalize this as follows. Let $f(R)$ be a weakly concave function, i.e. $f'(R) > 0, f''(R) \leq 0$. Then let the number of customers who can use the rebate to solve their liquidity problems be $\alpha(1-c_0)f(R)$. Specifically, the term $\alpha(1-c_0)$ represents the number of buyers who are initially unable to buy a car because they lack the adequate liquidity while $f(R)$ represents the percentage of these consumers who now can meet the liquidity constraint because of the rebate.

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3 Note that consumers could also sell their car and go without car transportation. However, we assume our parameters are such that this option is dominated by the other two options.
We first determine how many consumers who have the ability to buy a new car find it better to buy a new car versus keeping their existing car. We do this by equating Equations 1 and 2 and solving for \( \theta \). This yields the following

\[
\theta_i = \frac{(p_n - R - p_u)}{n(\gamma) - u(\gamma)}.
\]

(3)

Then consumers with \( \theta_i > \theta_i \) find it best to buy a new car and those with \( \theta \) values less than \( \theta_i \) finding it best to keep their old car (see Figure 2). Thus, all else equal, increases in \( R \) (i.e., the cash rebate) will decrease \( \theta_i \), i.e., shift it to the left, thereby increasing the number of new car buyers. Of course, not all else will be equal since both the optimal value for retail price \( p_n \) and the market clearing used car price \( p_u \) could change with \( R \), i.e., the retailer could react by increasing \( p_n \) and the increased supply of used cars due to the increased sale of new cars could decrease \( p_u \). Both effects shift \( \theta_i \) to the right, thereby reducing the number of new car buyers.

We next look at consumers who do not currently own a useable car. (Conceptually these consumers are entering the market for the first time, own an old car that has fully deteriorated, etc.). We assume the following for this segment.

1) Consumers are distributed uniformly with respect to their valuations over the full range of the unit line, i.e., \( \theta \leq \theta_i \leq \tau \).

2) All consumers have the ability to make the down payment.\(^4\)

Consumers in this segment now have three options, buy a new car, buy a used car, or remain inactive. The three utilities for customer \( i \) are respectively,

\[
u^2_{ni} = n(\gamma)\theta_i - p_n + R ,
\]

(4)

\[
u^2_{ui} = u(\gamma)\theta - p_u ,
\]

(5)

\[
u^2_{li} = 0 ,
\]

(6)

where the \( u \) and \( l \) subscripts represent buying a used car and remaining inactive respectively and the superscript 2 denotes the consumer segment that does not own a useable car at the start of the period.

\(^4\) We could easily relax this assumption, but it would only add to the complexity of the model and not provide any new insights. Thus, one way of thinking about our setup is that customers who lack the ability to pay are those individuals who are “upside down” with their currently owned car.
Equating Equations 4 and 5 yields the indifference point between the group that buys a new car and the group that buys a used car. Interestingly, this indifference point is equal to $\theta_1$, i.e., the same as the one for the segment owning a car. Equating Equations 5 and 6 yield the indifference point between buying a used car and remaining inactive. This latter indifference point, which we denote as $\theta_2$, is equal to $\frac{p_u}{u(\gamma)}$ (See Figure 2).

We now have enough information to write out the demand for new cars and the supply and demand for used cars. The demand for new cars comes from three different customer groups. We quantify this as follows. The number of customers who currently own car and have the ability and willingness to buy a new car at prices $p_n$ and $p_u$ before any rebate is $\alpha(c_0) (\tau - \theta_1)$. The number of customers who are given the ability to buy because of the rebate is $\alpha(1-c_0)f(R)$. After accounting for willingness, the demand for this group is $\alpha(1-c_0)f(R)(\tau - \theta_1)$. Finally, the demand for new cars from the consumers in the segment which starts out without useable cars is $(1-\alpha) (\tau - \theta_1)$. Combining these three groups of customers yields

$$q_n = \alpha(c_o + (1-c_o)f(R))(\tau - \theta_1) + (1-\alpha)(\tau - \theta_1),$$

(7)

where $S = \alpha(c_o + (1-c_o)f(R)) + (1-\alpha)$, $b = \frac{1}{n(\gamma) - u(\gamma)}$, and $T = [\tau((n(y) - u(\gamma)) - p_u + R)b$. Equation 7 is our derived demand curve and thus is one of the underlying drivers of our results.

We note that the size of the rebate has two different effects. Increases in $R$ increase $S$ (since $f(R)$ is assumed to be an increasing function in $R$) as well as $T$ (assuming $p_u$ remains fixed). Thus the term $ST$ shifts the demand function outward while the term $Sb$ makes the demand function steeper in own price, $p_n$.

We acknowledge that used car prices are endogenous and thus are a function of the exogenous actions of our model and of our model parameters. We derive these prices by equating the supply and demand of used cars. The supply of used cars comes directly from the segment of size $\alpha$ of previous car owners who trade in their cars to buy a new car, i.e.,

$$s_u = \alpha(c_o + f(R)(1-c_o))(\tau - \theta_1).$$

(8)

The demand for used cars can be determined from the segment of size $(1-\alpha)$ of consumers who now don’t own a useable car and find it best to buy a used car, i.e.,
Equating Equations 8 and 9 and substituting for $\theta_1$ and $\theta_2$ yields used car prices as a function of new car prices and the cash rebate as well as our four model parameters, $\alpha$, $c_0$, $\tau$, and $\gamma$. We then substitute this used car price equation into Equation 7 yielding:

$$p_u = \frac{u(\gamma)[(p_n - R)(1 - (1 - c_0)\alpha) - c_0c_\alpha(n(\gamma) - u(\gamma)) + (1 - c_0)c_\alpha f(R)(p_n - R - \tau(n(\gamma) - u(\gamma))]}{(1 - \alpha)n(\gamma) + \alpha(c_0 + (1 - c_0)f(R))u(\gamma)}.$$  

Substituting Equation 10 into 7, we get new car demand after taking into account the endogenous used car prices, i.e.,

$$q_n = \frac{(1 - \alpha)(1 - \alpha + c_0\alpha + (1 - c_0)c_\alpha f(R))(\tau n(\gamma) - p_n + R)}{(1 - \alpha)n(\gamma) + \alpha(c_0 + (1 - c_0)f(R))u(\gamma)}.$$  

Before using Equation 11 to derive equilibrium prices and quantities, we briefly summarize our approach. We want to reflect the fact that the cash rebate increases the pool of available buyers. However, we also want to capture the fact that this increase is not costless. Specifically, increasing the sale of new cars also increases the supply of used cars and thus, lowers the price of these cars. Since used cars compete with the sale of new cars, it is possible that there exists some upper bound on how much of a cash rebate the manufacturer would want to give. Our development yields a demand function which is directly linked to these two opposing forces, i.e., expanding the pool of potential buyers and lowering used car prices. The net result is a demand structure where the slope and intercept are a complicated function of our four model parameters.

4. ANALYSIS

We make the usual assumption that the manufacturer is the Stackelberg leader, setting both the wholesale price and the size of the per car cash rebate given to the consumers who buy their new car, after taking into account the profit maximizing actions of the dealer. The dealer takes as given the wholesale price and the size of the rebate and sets a retail price so as to maximize his retail profit. The retailer’s profit function is then $\pi_R = (p_n - w_n)q_n$, where $w_n$ is the manufacturer’s wholesale price for new cars. In calculating manufacturer profit we assume the only marginal costs for the manufacturers are the customer rebate, i.e., $R$, and the
administrative costs associated with the rebate. We define the latter to be \( kR \) where \( k>0 \). Thus, the manufacturer’s profit function is \( \pi_{M} = (w_{m} - kR_{n} - R)q_{m} \). Using the demand function stated in Equation 11 and the rules of the game stated above we are able to determine retail and wholesale prices, the magnitude of the cash rebate given and manufacturer and retailer profits. The results are given in Table 1.

4.1 Key Results

We first examine if the manufacturer finds it optimal to offer cash rebates. Our underlying model has cash rebates expanding demand by shifting the demand function outward as well as sliding down the demand curve via an effective lower price. Equation 7 makes it clear that the shift outward occurs only if \( \alpha > 0 \) and \( c_{0}<1 \). Consequently, it is not surprising that Lemma 1 indicates that the manufacturer does not always find it profitable to give customer rebates i.e., :

**Lemma 1:** Let \( R^{*} \) be the optimal rebate given to a buyer by the manufacturer. Then \( R^{*} > 0 \) only for only a subset of our parameter values \( \alpha, c_{0}, \gamma, \tau, \) and \( k \).

Specifically we find the manufacturer offers a cash rebate only when

\[
-2k(1-(1-c_{0})\alpha)(n(\gamma) - \alpha n(\gamma) - \alpha c_{0}u(\gamma)) + (1-c_{0})(1-\alpha)\alpha \tau n(\gamma)(n(\gamma) - u(\gamma))f'(R = 0) > 0.
\]

From this condition, it is clear that when the cost of offering promotions, \( k \), is low, the fraction of upside-down consumers \( (1-c_{0}) \) is high, and the effectiveness of the rebate in solving the upside problem, as represented by \( f'(R) \), is high, the manufacturer is more likely to offer the cash rebates.

We next explore what happens to prices, quantities and profits when the manufacturer offers a rebate. We note that the rebate allows some upside down consumers to replace their used cars with new cars, potentially increasing new car sales. However, this influx of high valuation customers into the market has two effects that could temper this sales increase. First, retailers may react to the outward shift of the demand curve and increase retail prices. Second, any increase in the supply of used cars will result in a decrease in used car prices, thereby,

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5 All proofs are in the Appendix.
increasing the competition between the two substitutable products. Manufacturers influence all this via their wholesale price and the size of the rebate.

The net result of this complex interact is as follows.

**Proposition 1:** In situations where the manufacturer finds it best to give a rebate, i.e., \( R^* > 0 \), the new car price, \( p_n^* \), and the effective price, \( (p_n^* - R^*) \), are greater than the retail price charged without a rebate. Interestingly even though the effective price increases, new car sales increase and used car prices decrease with the rebate.

**Proposition 2:** In situations where the manufacturer finds it best to give a rebate, i.e., \( R^* > 0 \), the manufacturer increases its wholesale price more than the amount of the cash rebate, but not enough to cover all of its marginal costs. Thus, the manufacturer’s margin decreases. Likewise the retailer is not able to raise its retail price enough to cover the increase in the wholesale price. Thus the retailer’s margin decreases. However, both parties’ profits increase over the situation where the manufacturer does not give a rebate.

Propositions 1 and 2 provide a detailed description of each agent’s actions. Two factors lead the retailer to increase its price. First, the manufacturer’s rebate reduces the effective retail price for consumers. The retailer takes advantage of this by increasing his retail price. Second, the manufacturer, sensing the shifting demand curve and the increased marginal costs associated with the rebate, increases its wholesale price, thereby driving up the retailer’s marginal cost. The net result is that the retailer’s margins decrease even though the effective retail price exceeds the price without rebate. However, since the quantity sold also increases with the rebate, the net result is that the retailer’s profits increase.

The rebate has mixed impact on consumers. Upside-down consumers are better off with the rebate because their liquidity constraints are relaxed and thus they can now buy a new car thereby increasing their net utility. However, all the other consumers who could have bought a new car without the rebate are worse off. Some of these consumers buy the new car but end up paying a higher price even after receiving the rebate. Others do not buy a new car, but instead either keep their used car or buy a used car. Thus, as with many marketing actions, there are “winners” and “losers”.

Finally, we note the counter balancing effects of higher effective retail price and the shifting outward of the demand function. The higher effective price causes some of the liquid consumers to not buy a new car, thereby shifting the identity of the marginal new (and thus used) car buyer to the right toward 1 (see Figure 2). On the other hand, the rebate allows some non-liquid consumers to buy a new car, thereby increasing the stock of used cars that need to be sold. This has the opposite effect, i.e., it shifts $\theta_2$ farther to the left of $\Theta_2$ in Figure 2.

We next study how the durability of the car, $\gamma$, affects the manufacturer's cash rebates decisions. As we discussed in the model section, the durability of the car represents how well the car holds up with usage. We previously made the realistic assumption that $n(\gamma) \geq u(\gamma)$, i.e., the value of a new car (for a fixed level of durability) is greater than or equal to the used car value. Moreover by definition the valuations are equal only when $\gamma=1$, i.e., the car is so durable that it does not deteriorate. These assumptions are compatible with the assumption that $u'(\gamma) > n'(\gamma)$, i.e., the used car valuation function increases faster than the new car valuation function, resulting in equality when $\gamma=1$. In words, this means that small changes in durability have a greater impact on the changes in valuation of a used car than a new car. For example a new car might have a valuation of $\gamma$ and the used car a valuation of $\gamma^2$ where $\gamma$ is restricted between $\frac{1}{2}$ and 1. Then $u'(\gamma) = 2\gamma$ and $n'(\gamma) = 1$ and thus $u'(\gamma) > n'(\gamma)$ since $\frac{1}{2} < \gamma < 1$. In the subsequent development we assume a somewhat more stringent condition concerning the durability parameter's effect on changes in the new and used car valuations. Specifically we assume:

\[(C1) \quad u'(\gamma) \geq n'(\gamma)[1 + \frac{\tau(n(\gamma) - u(\gamma))}{\tau n(\gamma) - k R}].\]

We note that (C1) is a sufficient condition for our proofs and is stronger than a necessary condition. In other words, our following results may hold even when this condition is not satisfied. However, for the remainder of this section, we assume that the condition in C1 holds. Using C1, we show the following two Propositions.

**Proposition 3:** The range of parameter values over which the optimal rebate, $R^*$, is positive, and therefore the likelihood that the manufacturer will give a cash rebate, decreases with increases in the durability of the car, $\gamma$. 

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Proposition 4: \(
\frac{\partial R^*}{\partial \gamma} < 0.
\)

In words, Propositions 3 and 4 state that a manufacturer with a more durable (and thus a higher quality) product is less likely to give a rebate and when it does give a rebate, it provides a smaller rebate than a manufacturer with a less durable product. The reason for this can be traced back to our underlying demand function and specifically to the effect of used car prices on the demand for new cars. From Equation 7 we note that the coefficient for \( p_u \) is:

\[ b = \frac{I}{n(\gamma) - u(\gamma)}. \]

By assumption \( b \) increases with \( \gamma \) since the difference in valuation of a new car and used car decreases with \( \gamma \). This implies that used cars and new cars become closer substitutes with increases in \( \gamma \). Consequently holding on to the existing car becomes a more attractive option for consumers who already have used cars and are contemplating replacing their used car with a new car. This means the number of consumers willing to trade in the used car and purchase a new car decreases with \( \gamma \). Since the purpose of cash rebates is to enable consumers who are willing (but unable) to replace their used car and buy a new car, the rebate becomes less effective. Said less technically, since the low durability manufacturer does not experience as much competition from its used car as does the high durability firm, it is able to get a bigger bang for its buck..

We noted earlier that the condition in (C1) is a strong sufficient condition. We say this because it is easy to find parameters for which our results are valid even when this condition doesn’t hold. Consequently in the next section we empirically test Propositions 3 and 4 to ascertain its external validity. In order to do so, we need to reconcile the fact that our model assumes the firm gives a rebate throughout the period, yet empirically firms seem to periodically give rebates. Other models (e.g., Lal, (1990), Rao (1991), etc.) address this issue by allowing the firm to use a mixed strategy, i.e., promote only with some probability. Our approach is somewhat different, but still captures the spirit of periodic rebates. Our model assumes the manufacturer determines the proportion of non-liquid consumers it wants to make liquid via the cash rebate. We capture this proportion via our parameter \( f(R) \), i.e., as \( R \) increases so does this proportion. We suspect there exists some psychological threshold in terms of the size of the rebate that will be noticed by consumers in order to generate some response. For example, we
suspect a constant offer of a $75 cash rebate for a $20,000 car would not attract much attention. However, an offer of $865 (=75*12) for one month out of a 12 month period would be noticed. Moreover, it would yield the ‘same’ effect as predicted by our model assuming the response is linear since under this linear assumption one month at $865 is equivalent to 12 months at $75.

Thus, although our model assumes a continuous promotion and a depth rate \( \frac{R^*}{p_n} \) that occurs throughout our one period model, we would not be surprised to empirically find firms using a depth rate \( \frac{R^{**}}{p_n} \) which is set to ‘catch’ consumers’ attention (thereby getting the desired response) but altering the length of time that they promote so that the effective average rebate rate, \( \frac{R^*}{p_n} \), is \( \frac{R^{**}}{p_n} \) multiplied by the proportion of time the firm promotes. We use this effective average rebate rate when we test Proposition 2.

5. EMPIRICAL ANALYSIS

Our theory offers several testable hypotheses. In this section, we test three as well as one of our underlying assumptions. The three hypotheses are as follows:

**H1:** The probability that consumer promotions are given decreases with the durability of the product.

**H2:** The depth of a consumer promotion, averaged over the model year, is higher for products with lower durability.

**H3:** The retail price of the new car increases with the depth of the consumer promotion and the durability of the car.

Hypothesis H1 comes directly from Proposition 3. H2 comes from Proposition 4 and our interpretation that it is equivalent for a firm to promote $x for the period of analysis or $nx for 1/nth of the period. H3 comes from Proposition 1 and Result 1.

5.1 Data Description and Measures

Our primary data source comes from a proprietary data base listing every incentive program offered by Daimler-Chrysler, Ford, GM, and the major Japanese (Honda, Mazda,
Nissan and Toyota) automobile manufacturers during 1992-1997 along with the duration of the promotion. In all, our database includes 69 nameplates (Toyota Camry, Ford Escort, etc) sold by these firms during the time period. These data were augmented by data that came from the following public sources: *Automotive News Market Data Book, Consumer Reports* and *Automotive Leasing Guide*. Since not every nameplate was sold in all six years, this resulted in a total of 317 nameplate/year observations. For each of these observations, we used the following measures where applicable:

**Consumer Promotion**: Our dependent measure for H1 is the indicator that a nameplate was on promotion in a given model year. We define a model year to be 16 months long. Since we are able to attach the time periods for specific promotion with a specific nameplate, there is no confusion in those periods where multiple model years are available, normally September through December.

**Retail Price**: We acknowledge that most retail transactions in the automobile industry are negotiated and thus are unique to the individual customer. Consequently, determining the actual average retail price is extremely difficult. We follow the lead of other researchers and use the Manufacturer’s Suggested Retail Price (MSRP) as a proxy for the average retail price, our dependent variable used in our analysis of H3.

**Average Consumer Cash Depth**: For every consumer promotion for a nameplate in a given year, we weight the per car value of the consumer promotion by the fraction of the year the car was on promotion and divide the result by the car’s MSRP in order to adjust for different average retail prices. (We replaced MSRP with the dealer’s invoice but this did not change our results.) If there were multiple consumer promotions per year, we then used the sum of these duration-weighted measures. Since we assume consumer depth is normally distributed (and thus conceptually varies from minus infinite to plus infinite), we rescale this weighted average proportion by taking the log odds of the weighted average.

**Durability**: Following the lead of Desai and Purohit (1999), we use the predicted annual reliabilities from Consumer Reports as our measure of durability. Predicted reliabilities are measured on a five-point scale for each nameplate and are based on the frequency of repair data for earlier vintages of the same model as well as Consumer Reports’ assessment of the durability of the current year’s model. Thus, it is a measure of durability that is a) available to consumers when making the buying decision, and b) based on both new car and used car durability.
Control Variables: One might postulate a number of other factors influence the frequency and depth of promotions as well as the retail price of the car. Consequently we next describe a number of variables we include in our analysis to control for such factors.

Degree of Product Differentiation: Our analytic model does not contain any inter-brand competition, yet some consumer promotions might be the result of competitive response. We control for any competitive effect by including a measure that captures the extent to which market shares are concentrated within the nameplate car segment. We divided the automobile market into segments using the scheme adopted by Automotive News Market Data Book. For each of these segments, we calculated the Herfindal index as: \( H_{it} = \sum m_{it}^2 \), where \( m_{it} \) is the market share of nameplate \( i \) in the year \( t \). Since that \( H_{it} \) increases as the market becomes more concentration this index is a good proxy for product differentiation. Basic economic theory predicts that the need to promote and the depth of promotions will decrease with product differentiation.

Average Monthly Excess Inventory: To control for the possibility that consumer promotions may be a response to inventory backlog, we first created a measure for the monthly changes in inventory levels. Specifically, for month \( t \), we use \( \Delta I_{it} = \frac{P_{it} - S_{it}}{S_{it}} \) where \( P_{it} \) is the production in month \( t \) and \( S_{it} \) is the sales in the same month for nameplate \( i \). This is mathematically equivalent to the increase (decrease) in monthly inventory as a fraction of the current month’s sales. This reflects the industry practice of thinking about inventory in terms of number of days of sales (referred to as days supply by the trade) and also controls for different rates of sales. Since only positive value of \( \Delta I_{it} \) lead to excess inventory levels, we use as our measure of excess inventory the sum of positive values of \( \Delta I_{it} \) where the summation is taken over a model year for a given nameplate. Thus, model years with higher values of our measure indicate the model year had (at least at times) higher (excess) inventory levels compared to model years with lower values of our measure. The production and sales data for all calendar years in this study are available in the Automotive News Market Data Book. Our conjecture is

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6 We did not have the data on absolute levels of inventory. The change in inventory could be calculated from production and sales data that are publicly available.
that if inventory levels are high for a nameplate, the firm would be more likely to promote and would give deeper discounts.

**Car vs. Truck:** Since consumer promotion policies may depend in part on whether the nameplate is a car or a truck, we include a dummy variable to reflect the fact that the promotion was for a car.

**Foreign vs. Domestic:** We create a dummy variable to indicate if the nameplate was a foreign car. This is important because empirically Japanese nameplates have, on average, higher predicted durability than the domestic nameplates. By including the foreign dummy along with our durability measure we are able to control for other factors associated with the foreign cars such as management policy, that might serve as an alternate explanation.

### 5.2 Empirical model

We jointly test H1 and H2 by specifying and simultaneously estimating a two-equation system that models an automobile manufacturer’s decision to offer a consumer rebate and conditional on offering a rebate, the size (depth) of the promotion. Formally, we investigate H1 and H2 together using a random effects variant of the widely applied sample selection model. (See for example, Verbeek 1990; Zabel 1992; Veerbeek and Nijman, 1992).

We model the decision to offer a consumer deal with a random effects Probit. More specifically, we define $z_{it}^*$ to be the latent propensity of an automobile manufacturer to promote nameplate $i$ (Toyota Camry, Ford Escort, etc) in year $t$. As shown in Equation 12 below we let $z_{it}^*$ to be a function of the vector of explanatory variables (one of which is durability), $w_{it}$, idiosyncratic disturbance $u_{it}$, and time-invariant, random nameplate specific effect, $\eta_i$. Similarly, we model the average depth of the consumer deal ($y_{it}$) for a nameplate $i$ in year $t$ as a function of the vector of explanatory variables (one of which is durability), $x_{it}$, disturbance $\varepsilon_{it}$ and nameplate specific effect, $\alpha_i$ (See Equation 13 below). The coefficients ($\beta$, $\lambda$) are the parameters to be estimated. Note that we observe $y_{it}$ and $x_{it}$ if and only if the auto manufacturer promotes, which we denote by letting $z_{it} = 1$. Following standard practice, we assume the idiosyncratic error terms ($\varepsilon_{it}$, $u_{it}$) follow bivariate normal distributions with mean vector zero and covariance matrix, $\Sigma$. The conditional distribution ($\varepsilon_{it} | u_{it}$) is also stated for later use. (See Equation 14.) Finally, we assume nameplate specific random effects, $(\alpha_i, \eta_i)$ are bivariate normal, with zero means, standard deviations ($\sigma_\alpha$ and $\sigma_\eta$) and CDF $G(\alpha_i, \eta_i)$ and uncorrelated with the nameplate effects, i.e.
We allow the disturbance components, $\varepsilon_{it}$ and $u_i$, to be correlated via the parameter $\rho$. Thus, the estimable parameters in our model are the slope and intercept parameters contained in vectors $\beta$ and $\lambda$, the variance parameters $\sigma_\varepsilon$, $\sigma_\alpha$ and $\sigma_\eta$ and the correlation parameter, $\rho$. (We make the non-restrictive assumption that $\sigma_\varepsilon=1$ to insure the system is identified.) Lastly, we make the standard, but somewhat restrictive assumption that the observations on different nameplates are independent even though some of the nameplates in our data are marketed by the same automobile manufacturer. Mathematically,

$$z_{it}^* = \lambda'w_{it} + \eta_i + u_{it}$$

$$z_{it} = 1(z_{it}^* > 0)$$

and $y_{it}$, $x_{it}$ observed if and only if $z_{it} = 1$,

$$y_{it} = \beta'x_{it} + \alpha_i + \varepsilon_{it}$$

$$(\varepsilon_{it}, u_i) \sim N(0, \Sigma)$$ where

$$\Sigma = \begin{bmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon \\ \rho\sigma_\varepsilon & 1 \end{bmatrix}$$

and thus,

$$u_{it} | \varepsilon_{it} \sim N\left( \frac{\rho\varepsilon_{it}}{\sigma_\varepsilon}, (1 - \rho^2) \right)$$

We obtain estimates of our model parameters by maximizing the theoretical unconditional likelihood of the $i$th nameplate. We derive this likelihood function by first stating the contribution of the $i$th nameplate in year $t$ to the likelihood conditional on the unobserved random effects. We then integrate out these unobserved variables. This conditional likelihood is comprised of three expressions, the conditional probability that a firm offers a consumer deal, the distribution of the consumer depth and the probability the firm does not offer a consumer deal. Thus the theoretical unconditional likelihood of the $i$th nameplate is

$$L_i(\beta, \lambda, \Sigma) = \int \int \prod_{i=1}^{T_i} f(\varepsilon_{it}, u_{it} | \alpha_i, \eta_i) \, dG(\alpha_i, \eta_i)$$

where $T_i$ is the number of years of data. We estimate the parameters using the maximum simulated likelihood routine found in LIMDEP 8.0.

We test H3 using another system of equations. This time our dependent variable of interest is retail price. Specifically we estimate the following system:
Deal Depth = $a_0 + a_1 \text{durability} + \text{error}$

Retail Price = $b_0 + b_1 \text{durability} + b_2 \text{deal depth} + b_3 \text{control dummies} + \text{error}$

From H2 we hypothesize that $a_1$ will be negative, while H3 implies that $b_2$ will be positive. Conventional logic states that $b_1$ should also be positive. We estimate this system of equations using three stage least squares.7

5.3 Results

The results of our analysis pertaining to H1 and H2 along with the list of the explanatory variables used in our analysis are provided in Table 2. As predicted, the durability coefficient in both the promotion and depth equations is statistically significant and negative; that is, the likelihood of consumer promotion decreases with durability (H1) and the average depth of consumer promotion decreases with durability (H2). In addition, the coefficient on product differentiation is negative and significant in the promotion likelihood equation, suggesting that consumer promotions are more likely to occur when markets are more competitive, i.e., less differentiated. Note, however, that product differentiation does not affect the average depth of the consumer promotion.

The foreign car indicator is insignificant in both equations. We previously noted that the American nameplates on average have a lower durability level than the Japanese nameplates. However, by including the foreign car indicator, we are able to rule out the possibility that our durability results are due entirely to differences in the management practices of the Japanese and American carmakers. Said more directly, we find the hypothesized durability effects even after controlling for any differences in managerial practices.8 This provides us with more confidence concerning our assertion that lower durability causes manufacturers to use consumer promotions and give deeper average promotions.

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7 We use a slightly different measure for deal depth in this analysis. Specifically, since MSRP is our dependent variable measure, we did not want to include this measure on the right hand side of the equation. Consequently, we did not divide the per deal amount by this measure. Instead we included a number of measures pertaining to the type of car being sold, i.e., luxury, SUV etc. to capture the fact that deal depth may be related to the initial price of the car.

8 Interestingly, our data shows that when foreign firms offer cash rebate, they offer on average higher rebates, but over shorter durations.
Another possible explanation for cash rebates is that these promotions are a way to manage inventory levels. Our Table 2 results provide some support for this belief, at least in terms of the likelihood that a firm offers a consumer promotion during the 16 months period. However, we find no impact of excess inventory levels on the depth of the promotion.

We further explore the impact of inventory levels on consumer promotions by looking at the frequency of consumer promotions by month over the 16 months associated with a given model year by aggregating the data across models and model years. Our analytic model assumes consumer promotions are offered throughout the period in order to solve consumers’ upside down problem. In contrast, the hypothesis that consumer promotions are used to clear inventories should show a different pattern since excess inventories historically exist at the end of the model year.

The results of our analysis for domestic and foreign nameplates are shown in Tables 3 and 4. There are several important observations. First, consumer promotions are given throughout the total model year. However, promotion frequency for foreign manufacturers is somewhat stronger in the latter part of the model year. We infer from this that inventory management may play a greater role in foreign firms’ decisions to offer consumer promotions. This is compatible with the observation that their supply chain is longer (i.e., most Japanese cars are shipped from Japan). In summary, although firms are more likely to use consumer promotions if there exists some inventory imbalance (especially among foreign nameplates), we find strong evidence that these promotions are used throughout the model year. This, in turn, supports our underlying model assumption and thus our Table 2 results.

We find the highly significant negative estimate of $\rho$ to be interesting. It suggests that when consumer promotions are given for causes not included in our model (and thus by assumption these causes are random), the resulting promotion has a smaller average depth. Although we have little evidence on why this occurs, this observation has strong face validity. In other words, if a firm gives a promotion for some unplanned reason, the average consumer depth will be lower than those consumer promotions that are planned.

The results of our test of H3 are found in Table 5 along with a list of control variables. As can be seen from these results, the coefficients for durability and deal depth are positive and significant even after adjusting for car type. Thus, compatible with Proposition 1, we find our proxy for dealers’ retail price increases when the manufacturer gives deeper consumer rebates.
Moreover, this finding occurs even after controlling durability and the type of car being sold. As predicted from H2 and Result 1, we find (again) that durability has a negative and significant effect on deal depth and a positive effect on retail price. We find these results encouraging, since they provide one more test of our model predictions.

6. SUMMARY AND CONCLUSIONS

The overarching goal of this paper is to explain why durable manufacturers might want to give rebates directly to consumers even when they cannot set the retail price. We do this by developing a model that reflects the fact that the viability of this pricing action depends on being able to shift the demand curve outward (versus sliding down the demand curve). This led us to relax the standard assumption that consumers always have the ability to pay for a product and thus only consumers’ willingness-to-pay for a product matters. We note that at least in the automobile industry, a significant percentage of the population lacks the liquidity to make the required down payment, i.e., lacks the ability to pay.

Our model makes an explicit link between the amount of cash rebate given (which can be used for the down payment) and the number of customers who will ultimately be able to buy. It also reflects the fact that new and used cars are partial substitutes. Thus, increases in new car sales to current owners result in an increased supply of used cars and ultimately lower used car prices. Since used cars are partial substitutes for new cars, the lower used car prices put a downward pressure on new car prices.

We show that there are situations where manufacturers find it profitable to provide the consumer with a cash rebate and the incidence of these situations increases for low durability manufacturers, i.e., manufacturers whose car deteriorates faster than average. Moreover, the depth of these promotions is larger for these lower durability manufacturers. We also show that retail prices increase with the deal depth. We test these three findings using a large data set of customer promotions given over the years 1992-1997. We find strong support for all three of our model predictions. We note that our analytic model makes a number of other predicts. Three of the most interesting are that retail and wholesale margins decrease with the rebate, retail profits increase when the rebates are given and the effective price paid by consumers increases with the rebate.
We note in passing that another way for the manufacturer to address the liquidity problem is by giving a discount to the trade and then relying on the dealer to pass this promotion on to the consumer. Such a procedure, however, is much less effective for two reasons. First, it is well known that the dealer will not pass on all of the promotion to the consumer. Second, although consumers see a decrease in retail price, this decrease does not materially lower the down payment requirements associated with buying a new car. Consequently trade promotions are a very ineffective mechanism for solving consumers’ liquidity problem and therefore shifting the demand function outward.

We also note that our model makes a few limiting assumptions. First, it is a one period model, and thus, does not reflect the possible interplay between giving cash rebates and the proportion of customers who may be unable to pay. One of the major reasons why customers become upside down is that their car depreciates faster than the customer makes monthly payments. We find cash rebates lead to lower used car prices, i.e., causes the car to depreciate (but not deteriorate) faster. This implies a linkage between the rebate size, \( R \), and the proportion of consumers who are upside down, \( c_\theta \), something not reflected in our model. We believe including this linkage would just increase the frequency of subsequent promotions. However, it should not alter our results qualitatively.

A second possible limitation is our decision not to model inter-brand competition. However, we again believe this decision should not affect the qualitative aspects of our conclusions. Assuming the competing durables being studied are only partial substitutes, there still should be a segment that finds each manufacturer’s brand most desirable. Consequently, each manufacturer would still want to solve the liquidity problem for its set of buyers. We noted in our empirical analyses that firms in a more competitive environment were more likely to give rebates, but not offer deeper rebates. Perhaps this reflects the firm’s attempt to impede defection of its customer base to a competitor who is offering cash rebate (versus helping out a subset of their current owners who otherwise couldn’t make the down payment).

We also acknowledge that our results are driven in large part by our (unverified) assumption that used car valuation is a faster increasing function than new car valuation. Although we provide no empirical evidence to support this assumption, we feel justified in making such an assumption since it is consistent with the concepts that a) new car valuation (for
a fixed level of durability) is greater than used car valuation and b) the two valuations are equal only when the new car never deteriorates, i.e., $\gamma$ approaches its upper limit.

Finally, we reemphasize our belief that the practice of providing customer rebates has less to do with adjusting inventory levels or lowering the effective retail price and more to do with the general liquidity of the current market place. Thus, if all consumers have the ability to pay, we would not expect durable manufacturers to offer customer rebates. (They might, however, offer promotions through the trade.) Likewise, if the percentage of customers who are upside down rises, we would expect the incidence of rebate to increase. This linkage to the proportion of customer’s who are unable to pay implies that we would only see gradual changes over time in rebate rates and depth. This observation is consistent with our data.
References


Figure 1

Distribution of Consumers who Own a Useable Car

Have necessary funds

Lack necessary funds

Ability to Pay

Valuation

$\theta_0$

$\tau$

$c_0$
Figure 2

Partitioning of Consumers

(a) Consumers who own a useable car

\[
\begin{array}{c|c}
\text{Ability to Pay} & \text{Valuation} \\
\hline
\theta_0 & \theta_1 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
\text{Keep old car} & \text{Buy new car} \\
\hline
\text{Keep old car} & \text{Buy new car} \\
\hline
\text{Keep old car} & \text{Keep old car} \\
\hline
\end{array}
\]

(b) Consumers who do not own a useable car

\[
\begin{array}{c|c|c}
\text{Valuation} & \theta_0 & \theta_1 \\
\hline
\text{Stay inactive} & \theta_0 & \theta_1 \\
\text{Buy used car} & \theta_0 & \theta_1 \\
\text{Buy new car} & \theta_0 & \theta_1 \\
\hline
\end{array}
\]
Table 1  
Equilibrium Values

\[ p_u = \frac{u(\gamma)[kR^*(1 - \alpha + \alpha c_0) + (3 - 3 + c_0)\alpha m(\gamma) + 4c_0\alpha n(\gamma) - (1 - c_0)\alpha f(R^*)(m(\gamma) - kR^* - 4n(\gamma))]}{4(1 - \alpha)n(\gamma) + 4\alpha(c_0 + (1 - c_0)f(R^*))u(\gamma)} \]

\[ w_n^* = \frac{R^*(2 + k) + m(\gamma)}{2} \]

\[ p_n^* = R^* + \frac{kR^*}{4} + \frac{3m(\gamma)}{4} \]

\[ \pi_r = \frac{(1 - \alpha)(1 - \alpha + \alpha(c_0 + (1 - c_0)f(R^*))(m(\gamma) - kR^*))^2}{16[(1 - \alpha)n(\gamma) + \alpha(c_0 + (1 - c_0)f(R^*))u(\gamma)]} \]

\[ \pi_m = \frac{(1 - \alpha)(1 - \alpha + \alpha(c_0 + (1 - c_0)f(R^*))(m(\gamma) - kR^*))^2}{8[(1 - \alpha)n(\gamma) + \alpha(c_0 + (1 - c_0)f(R^*))u(\gamma)]} \]
### Table 2

#### Estimates of Empirical Models for Hypotheses H1 and H2

<table>
<thead>
<tr>
<th>Models:</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Promotion Likelihood:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.1659*</td>
<td>0.0941</td>
<td>9.523</td>
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<tr>
<td>Durability</td>
<td>-0.2912*</td>
<td>0.0726</td>
<td>-4.010</td>
</tr>
<tr>
<td>Herfindahl Index</td>
<td>-3.7705*</td>
<td>1.0414</td>
<td>-3.620</td>
</tr>
<tr>
<td>Foreign/Domestic</td>
<td>-0.1898</td>
<td>0.1803</td>
<td>-1.084</td>
</tr>
<tr>
<td>Excess Inventory</td>
<td>1.3528*</td>
<td>0.5223</td>
<td>2.590</td>
</tr>
<tr>
<td>Vehicle Type</td>
<td>-0.3727*</td>
<td>0.1803</td>
<td>-2.068</td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>1.1659*</td>
<td>0.0940</td>
<td>12.399</td>
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<tr>
<td><strong>Average Consumer Deal Depth:</strong></td>
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<tr>
<td>Constant</td>
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<td>0.0607</td>
<td>-8.261</td>
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<tr>
<td>Durability</td>
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<td>0.0190</td>
<td>-5.226</td>
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<td>Herfindahl Index</td>
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<td>Foreign/Domestic</td>
<td>0.0408</td>
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<tr>
<td>Excess Inventory</td>
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<tr>
<td>( \sigma_\alpha )</td>
<td>0.1473*</td>
<td>0.0124</td>
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<tr>
<td>( \sigma_\varepsilon )</td>
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<tr>
<td>( \rho )</td>
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<td>LL</td>
<td>-278.62</td>
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Sample Size (N=317) Panels (69 Nameplates)

*: significant at 1% significance level

**: significant at 5% significance level
Table 3

Aggregate Frequency Distribution of Consumer Promotion Over the Model Year
(16 month period)

Domestic Cars

<table>
<thead>
<tr>
<th>Month</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Frequency</th>
<th>Cumulative Percent</th>
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<td>1</td>
<td>184</td>
<td>5.68</td>
<td>184</td>
<td>5.68</td>
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<tr>
<td>2</td>
<td>200</td>
<td>6.18</td>
<td>384</td>
<td>11.86</td>
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<td>3</td>
<td>203</td>
<td>6.27</td>
<td>587</td>
<td>18.13</td>
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<td>4</td>
<td>203</td>
<td>6.27</td>
<td>790</td>
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<td>201</td>
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<td>6.30</td>
<td>1399</td>
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<td>207</td>
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<td>1814</td>
<td>56.04</td>
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<td>207</td>
<td>6.39</td>
<td>2021</td>
<td>62.43</td>
</tr>
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### Table 4

Aggregate Frequency Distribution of Consumer Promotion Over the Model Year  
(16 month period)

**Foreign Cars**

<table>
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<tr>
<th>Month</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Frequency</th>
<th>Cumulative Percent</th>
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<td>36</td>
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<td>5.81</td>
<td>150</td>
<td>24.19</td>
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<td>6</td>
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<td>187</td>
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<td>7</td>
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### Table 5
Three-Staged Least Square Results, Model for H3

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<th>Standard Error</th>
<th>t-stat</th>
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<tr>
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<td>12.2404</td>
<td>-2.495</td>
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<tr>
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<td>3.013</td>
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<td>Log(Deal)</td>
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<td>1.5962</td>
<td>3.286</td>
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<td>0.1827</td>
<td>0.227</td>
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<tr>
<td>Pickup</td>
<td>0.2522</td>
<td>0.1405</td>
<td>1.795</td>
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<tr>
<td>SUV</td>
<td>1.3584**</td>
<td>0.5917</td>
<td>2.296</td>
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<tr>
<td>Mid-Sized</td>
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<tr>
<td>Std. Dev. = 4.244</td>
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<td></td>
<td></td>
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<tr>
<td>RSS = 4467.894</td>
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<tr>
<td><strong>Dependent Variable:</strong> Log (Deal):</td>
<td></td>
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<td></td>
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<tr>
<td>Constant</td>
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<td>Durability</td>
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<td>Std. Dev. = 0.821</td>
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<tr>
<td>RSS = 177.16</td>
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</tbody>
</table>

Sample Size (N=256)

*: significant at 1% significance level

**: significant at 5% significance level
Appendix

Proof of Lemma 1:

After substituting for \( p_u^* \), the new car demand function is given by:

\[
q_n = \frac{(1-\alpha)(1-\alpha + c_0 \alpha + (1-c_0) \alpha f(R)) (\tau n(\gamma) - p_n + R)}{(1-\alpha)n(\gamma) + \alpha(c_0 + (1-c_0)f(R))u(\gamma)}, \text{ where } 0 < f(R) < 1, f'(R) > 0
\]

and \( f''(R) < 0; \ n(\gamma) \geq u(\gamma), n'(\gamma) > 0, \text{ and } u'(\gamma) > 0 \). We can now determine both the retailer’s and manufacturer’s optimal decisions when rebates are offered:

The retailer’s profit, \( \pi_r \), is given by \( \pi_r = (p_n - w_n)q_n \). Solving the retailer’s optimization problem, we get \( p_n^* = \frac{R + n(\gamma)\tau + w_n}{2} \). The manufacturer’s profit is then given by:

\[
\pi_m = \frac{(1-\alpha)(w_n - R - kR)(n(\gamma)\tau + R - w_n)(1-\alpha + \alpha(c_0 + (1-c_0)f(R)))}{2((1-\alpha)n(\gamma) + \alpha(c_0 + (1-c_0)f(R))u(\gamma))}
\]

The first-order condition for \( w_n \) is:

\[
(1-\alpha)((2+k)R - 2w_n + n(\gamma)\tau)(1-\alpha + \alpha(c_0 + (1-c_0)f(R))) = 0, \text{ giving } w_n^* = \frac{(2+k)R + n(\gamma)\tau}{2}.
\]

Similarly, the first-order condition for rebate, \( R \), evaluated at the optimal wholesale \( w_n^* \) price is:

\[
\frac{\partial \pi_m}{\partial R} \bigg|_{w=w_n^*} = -2k(1-\alpha + x_1)((1-\alpha)n(\gamma) + x_1u(\gamma)) + x_2(\pi n(\gamma) - kR)(n(\gamma) - u(\gamma))f''(R) = 0,
\]

where \( x_1 = \alpha c_0 + \alpha(1-c_0)f(R), \ x_2 = \alpha(1-\alpha)(1-c_0) \text{ and } A = \frac{8((1-\alpha)n(\gamma) + x_1u(\gamma))^2}{(1-\alpha)(\pi n(\gamma) - kR)} > 0 \).

Note that if \( R^* > 0 \) then \( \frac{\partial \pi_m}{\partial R} \bigg|_{w=w_n^*} \) evaluated at \( R=0 \) must be positive, i.e., the profit function must be increasing in \( R \). This partial derivative is negative for
\{τ = 2, f(R) = 0.2R, c_o = 0.2, α = 0.25, k = 0.0125, n(γ) = γ, u(γ) = γ^2, γ = 0.6\} and is positive for \{τ = 2, f(R) = 0.2R, c_o = 0.2, α = 0.25, k = 0.01, n(γ) = γ, u(γ) = γ^2, γ = 0.55\}.

**Proof of Proposition 1**

We denote the case when the manufacturer does not offer a rebate with the superscript \(**\). The retail price when the manufacturer does not offer a rebate, (i.e. when \(R = 0\)) is \(p_n^{**} = \frac{3m(γ)}{4}\).

Thus, \(p_n^* - p_n^{**} = R^* + \frac{kR^*}{4}\) and \(p_n^* - R^* - p_n^{**} = \frac{kR^*}{4}\). That is, new car price, \(p_n^*\), and the effective price, \((p_n^* - R^*)\), with rebate are greater than those without a rebate. We confirm that new car sales \(q_n^*\) increase and used car prices \((p_u^*)\) decrease with the rebate by first obtaining the following derivatives:

\[
\frac{∂q_n^*}{∂R} = -\frac{k(1-α + x_1)((1-α)n(γ) + x_1u(γ)) + x_2(m(γ) - kR)(n(γ) - u(γ))f'(R)}{B}
\]

\[
\frac{∂p_u^*}{∂R} = \frac{u(γ)[k(1-α + x_1)((1-α)n(γ) + x_1u(γ)) - x_2(m(γ) - kR)(n(γ) - u(γ))f'(R)]}{C}
\]

where \(x_1\) and \(x_2\) are as defined in the proof of Lemma 1, \(B = \frac{4((1-α)n(γ) + x_1u(γ))^2}{(1-α)} > 0\) and \(C = 4((1-α)n(γ) + x_1u(γ))^2 > 0\).

We note that at the optimal rebate, \(R^* > 0\), \(\frac{∂π_m}{∂R} \bigg|_{R = R^*, w = w^*_u} = 0\). This implies that

\[-2k(1-α + x_1)((1-α)n(γ) + x_1u(γ)) + x_2(m(γ) - kR)(n(γ) - u(γ))f'(R^*) = 0\]. Thus, it is easy to see that \(\frac{∂q_n^*}{∂R} > 0\) and \(\frac{∂p_u^*}{∂R} < 0\) at \(R^* > 0\).
Proof of Proposition 2:

The manufacturer’s wholesale price when \( R = 0 \) is \( w_n^{**} = \frac{\tau n(\gamma)}{2} \). Thus, the increase in wholesale prices due to the rebate is: \( w_n^* - w_n^{**} = R^* + \frac{kR^*}{2} > R^* \). It is easy to see that the manufacturer’s margin with the rebate is \( w_n^* - R^*(1 + k) = \frac{m(\gamma)}{2} - \frac{kR^*}{2} \), which is less than the manufacturer’s margin without the rebate, \( w_n^{**} = \frac{m(\gamma)}{2} \).

From the proof of Proposition 1 we note that the change in retail price when the manufacturer offers a rebate is \( p_n^* - p_n^{**} = R^* + \frac{kR^*}{4} \) which is positive but less than the increase in the manufacturer’s margins, \( R^* + \frac{kR^*}{2} \).

Finally note that \( R^* > 0 \) if and only if the manufacturer’s profit increases with the rebate, i.e., \( \pi_m^* > \pi_m^{**} \). To show that retailer profits are also higher when \( R^* > 0 \), we simply note that \( \pi_r^* = \frac{\pi_m^*}{2} \) and \( \pi_r^{**} = \frac{\pi_m^{**}}{2} \). Thus, the retailer’s profit increases compared to the case when the manufacturer does not give a rebate. \( \blacksquare \)

Proof of Proposition 3

\[
\frac{\partial \pi_m}{\partial R} \bigg|_{w=w_n^*} = \frac{-2k(1-\alpha + x_i)((1-\alpha)n(\gamma) + x_iu(\gamma)) + x_i(\gamma - kR)(n(\gamma) - u(\gamma))f'(R)}{A}
\]

where \( A \) is as defined in the proof of Lemma 1. Let \( \psi_i \) be the numerator of the above expression evaluated at \( R=0 \). Then,
\[ \psi_i = -2k(1 - \alpha + x_i)(1 - \alpha)n(\gamma) + x_iu(\gamma) + x_2(\gamma)(n(\gamma) - u(\gamma))f'(R = 0) \]. As discussed in the proof of Lemma 1, \( R^* > 0 \) only when \( \psi_i > 0 \), i.e., the profit function is increasing in \( R \).

It can be shown that \( \frac{\partial \psi_i}{\partial \gamma} = (e_1 + e_2 + e_3) \) where:

\[
e_1 = x_2(\gamma)(n(\gamma) - u(\gamma))f'(0)n'(\gamma) > 0 \]

\[
e_2 = x_2(\gamma)(n(\gamma))f'(0)(n'(\gamma) - u'(\gamma)) < 0 \text{ if } u'(\gamma) - n'(\gamma) > 0 \]

\[
e_3 = -2k(1 - \alpha + c_0(\gamma)(1 - c_0)(\gamma)(f'(0)(\gamma)(\gamma) + \alpha(1 - \gamma) + (1 - c_0)(\gamma)u'(\gamma)) < 0 \]

Note that, \( (e_1 + e_2) = x_2(\gamma)f'(0)[(2\tau(\gamma) - \tau u(\gamma))n'(\gamma) - (\tau(\gamma))u'(\gamma)] \leq 0 \) if

\[
u'(\gamma) \geq 1 + \frac{\tau(n(\gamma) - u(\gamma))}{\tau n(\gamma) - k R^*} \], which is our sufficient condition (C1). This implies that as \( \gamma \) increases, \( \psi_i \) is more likely to be negative and thus \( R^* \) is less likely to be positive.

Proof of Proposition 4

As shown in the proof of Lemma 1, after substituting for \( w_n = w_n^* \), the first-order condition for \( R \) is given by

\[-2k(1 - \alpha + x_i)(1 - \alpha)n(\gamma) + x_iu(\gamma) + x_2(\gamma)(n(\gamma) - u(\gamma))f'(R = 0) = 0 \].

We denote the LHS of the above equation by \( \psi_2 \). Note that \( R^* \) is implicitly defined by \( \psi_2 = 0 \). By the implicit function theorem, \( \frac{\partial R^*}{\partial \gamma} = -\frac{\partial \psi_2}{\partial \gamma}/\frac{\partial \psi_2}{\partial \gamma} \). First, \( \frac{\partial \psi_2}{\partial R} = -\alpha(1 - c_0)(j_1 + j_2) < 0 \) where:

\[
j_1 = kf'(R^*)(3(1 - \alpha)n(\gamma) + (1 - \alpha + 4c_0)\alpha + 4(1 - c_0)(\alpha f'(R^*)u(\gamma)) > 0 \]

\[
j_2 = -(1 - \alpha)(m(\gamma) - kR^*)(n(\gamma) - u(\gamma))f''(R^*) > 0 .\]
We can show that: \( \frac{\partial \psi_2}{\partial \gamma} = (e_4 + e_5 + e_6) \) where \( e_4 = x_2 \tau (n(\gamma) - u(\gamma)) f'(R^*)n'(\gamma) > 0 \), 

\( e_5 = x_2 (m(\gamma) - kR^*) f'(R^*) (n'(\gamma) - u'(\gamma)) < 0 \) if \( u'(\gamma) - n'(\gamma) > 0 \), and 

\( e_6 = -2k(1 - \alpha + c_0 \alpha + (1 - c_0) \alpha f(R^*))((1 - \alpha)n'(\gamma) + \alpha(c_0 + (1 - c_0)f(R^*))(u'(\gamma)) < 0 \).

Note that, \( (e_4 + e_5) = x_2 \alpha f(R^*)\left(2m(\gamma) - kR^* - pu(\gamma)\right)n'(\gamma) - (m(\gamma) - kR^*u'(\gamma)) \leq 0 \) if 

\( \frac{u'(\gamma)}{n'(\gamma)} \geq 1 + \frac{\tau(n(\gamma) - u(\gamma))}{\tau n(\gamma) - k R^*} \), which is our sufficient condition (C1). This implies that 

\( \frac{\partial R^*}{\partial \gamma} = -\frac{\partial \psi_2 / \partial \gamma}{\partial \psi_2 / \partial R} < 0 \). 

\[ \square \]